The Distance Learning Centre
LEARNING MATERIALS

Course: GCSE

Syllabus: AQA GCSE Mathematics

Subject: Mathematics

Theme: Number

Topic 2: Sample Materials

NOTE: Higher Tier

(HIGHER TIER)

When you see the words Higher Tier (as shown left) next to a heading, and a mauve tinted background to related sections of text, this denotes sections of the learning materials that you only need to learn, understand and revise if you are planning to sit the Higher Tier exams. If you are just planning to sit the Foundation Tier exams, you do not need to read or learn these sections.

Essential reading - FOUNDATION TIER:

The essential textbook for studying this topic at Foundation Tier is *AQA GCSE Maths Foundation Student Book* (2015). By Stephen Fearnley, June Haighton, Steven Lomax, Peter Mullarkey, James Nicholson and Matthew Nixon. Published by Oxford University Press. ISBN 13: 978-0-19-835165- 8.

Essential reading - HIGHER TIER:

The essential textbook for studying this topic at Higher Tier is *AQA GCSE Maths Higher Student Book* (2015). By Stephen Fearnley, June Haighton, Steven Lomax, Peter Mullarkey, James Nicholson and Matthew Nixon. Published by Oxford University Press. ISBN 13: 978-0-19-835166-5.

Essential reading: (FOUNDATION TIER TEXTBOOK)

You will find references to relevant sections of the Foundation Tier textbook, as you work through these learning materials. These are shown with a yellow highlight, and a blue tinted box, as shown here.

Essential reading: (HIGHER TIER TEXTBOOK)

You will find references to relevant sections of the Higher Tier textbook as you work through these learning materials. These are shown with a yellow highlight, and a mauve tinted box, as shown here.

Things to do: (FOUNDATION TIER)

You will find things to do from the Foundation Tier textbook as you work through the learning materials. These are shown with a plain text title (no highlight), and a cream tinted box, as shown here.

Things to do: (HIGHER TIER)

You will find things to do from the Higher Tier textbook as you work through the learning materials. These are shown with a yellow highlight in the title, and a mauve tinted box, as shown here.

Recommended reading:

Recommended reading for this topic is listed in the back pages of these learning materials.

Read and make notes:

Before starting work, we recommend that you read through all the content in this document, and make notes about the key information that you will be learning, anything that you don't understand and need to explore further, and the tasks that you will need to carry out.

Course: **GCSE**
Subject: **Mathematics**
Theme: **Number**
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Sample Contents Page

Course: **GCSE**
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LEARNING MATERIALS

2. Working with factors and multiples

Essential reading: (FOUNDATION TIER TEXTBOOK)

Foundation Tier textbook, section 13.1, page 272.

Essential reading: (HIGHER TIER TEXTBOOK)

Higher Tier textbook, section 13.1, page 256.

Ready, steady, go!

Now that we know what factors and multiples are, we are going to find out how to find the highest common factor and the least common multiple between sets of numbers.

2.1 Highest common factor

We have seen that any whole number can be written as the **product** of two factors:

$$84 = 4 \times 21$$

So 4 and 21 are **factors** of 84

Sometimes we want to consider two numbers and we want to find the **highest common factor** (hcf) of these numbers. This means that we are looking for the highest number that goes into both of the numbers. We can find this by listing all of the factors of both numbers.

Example 1: Highest common factor

- **Find the highest common factor of 12 and 18.**

First we list the factors of both numbers:

The factors of **12** are:	**1**	**2**	**3**	**4**	**6**	**12**
The factors of **18** are:	**1**	**2**	**3**	**6**	**9**	**18**

The common factors of 12 and 18 are the numbers that appear in both lists. These are:

1 2 3 6

The highest number that appears in both lists is 6. **So 6 is the highest common factor of 12 and 18.**

SAQ 2:

1. **Find the highest common factor of:**
   a) **12 and 24**
   b) **15 and 40**
   c) **32 and 48**
   d) **8, 24 and 42**
   e) **18, 24 and 36**

2.2 Least common multiple

So far we have seen how to work out multiples of numbers. Sometimes when we consider two numbers, we want to find the **least common multiple** of the two numbers. This means that when we list the multiples of the two numbers, we are looking for the first number to occur in both the lists. This will be the least common multiple.

Example 2: Least common multiple

- **Find the least common multiple of 12 and 18.**

This time we list the multiples of both numbers:

The multiples of **12** are:	**12**	**24**	**36**	**48**	**60**	**72**
The multiples of **18** are:	**18**	**36**	**54**	**72**		

Looking at both lists we can see that the common multiples are 36 and 72 (there will be many more). But we want the lowest number – the first one to appear in both lists. This is 36. So we can say that **36 is the least common**

multiple of 12 and 18.

SAQ 3:

1. **Find the least common multiple of:**
   a) **3 and 5**
   b) **8 and 12**
   c) **20 and 25**
   d) **6 and 8**

Things to do: (FOUNDATION TIER)

Work through the examples in section 13.1 on page 272 of the Foundation Tier textbook. Then complete exercise 13.1S on page 273, and exercise 13.1A on page 275.

2.3 Prime numbers

Things to do:

1. **On the table below, shade all the multiples of 2 except 2:**

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

2. **Now shade all the multiples of 3 except 3.**
3. **Shade all the multiples of 5 except 5 (all the multiples of 4 should already be shaded).**
4. **Shade all the multiples of 7 except 7 (all the multiples of 6 should already be shaded).**
5. **Your next un-shaded number should be 11. Shade all the**

multiples of 11 except 11.

6. **Keep going to the next un-shaded number and shade all the multiples of it, except the number itself.**

Eventually you will have no multiples left but you should still have some unshaded numbers. Look at these numbers. How many factors do they have?

The numbers you have left are special. They are called prime numbers. Each prime number only has two factors: itself and 1

It is important to remember that 1 is not a prime number.
The first few prime numbers are:

2 3 5 7 11 13 17 19 23 29 31 37 41 43 47

Important point:

You should try to learn all the prime numbers which are less than 20.

2.4 Prime numbers and factorisation

Essential reading: (FOUNDATION TIER TEXTBOOK)

Foundation Tier textbook, section 13.2, page 277.

We have already seen that we can write a number as a product of its factors. What we are going to look at now is how we write a number as a product of just **prime** factors, so we are going to break down numbers into a combination of prime factors. This is **prime factor decomposition**.

Remember that **a prime number only has two factors – itself and 1**. But all other numbers have more than two factors, and can be represented by a unique combination of prime factors.

Let's take the number 24 and consider any two numbers that you can multiply together to make that number. Let's consider 2 x 12.

Well we know that 2 is a prime number, but 12 isn't. So we would continue to divide this into 2 x 6.

Again, 2 is prime but 6 is not. So we continue and divide again. 6 = 2 x 3 and this time both

of the numbers are prime.

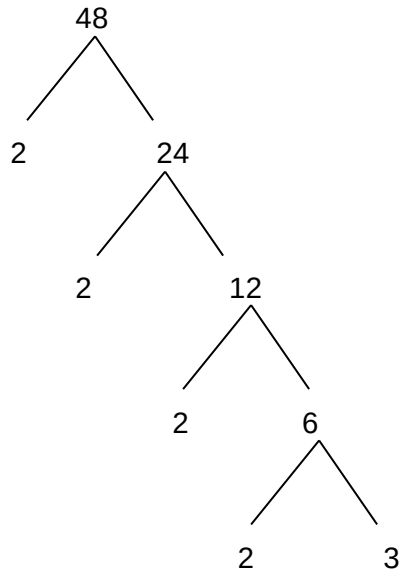
So we can say that **24 as a product of its prime factors is 2 x 2 x 2 x 3.** We can also write this as $\mathbf{2^3 \times 3}$. When you write it like this it is called **index form**.

To make sure that you find all the prime factors you should proceed in a systematic way. That way you can be sure that you have found all the factors.

2.5 Prime factor trees

Example 3: Using prime factor trees

- **Express 48 as a product of prime factors.**



1. Write down the number you want to factorise.
2. Choose a prime factor, and place it under the left branch.
3. Divide the original number by the factor and put this new number against the right branch.
4. Repeat the process with your new number.
5. When all numbers at the bottom of the branches are prime numbers, you are finished.

So 48 = 2 x 2 x 2 x 2 x 3
or $2^4 \times 3$

If you look at **page 162** in the textbook it describes another way to find the prime factors using repeated division. Either method is acceptable; just use the one you like best.

SAQ 4:

1. **Show each of the following numbers as a product of prime factors. Give your answer in index form:**
   a) **36**
   b) **34**
   c) **715**
   d) **825**
   e) **2,854**

We have already looked at how we can find the highest common factor (HCF) and lowest common multiple (LCM) of given numbers. Well now we can use the prime factors to find these.

2.6 Finding the HCF using prime factors

Essential reading: (FOUNDATION TIER TEXTBOOK)

Foundation Tier textbook, section 13.2, page 276.

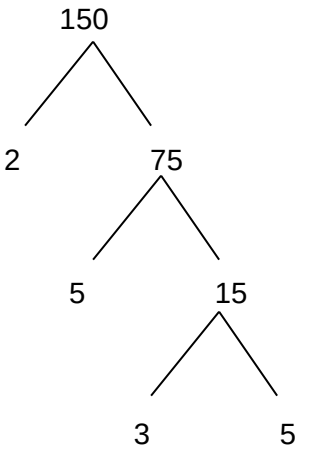
Essential reading: (HIGHER TIER TEXTBOOK)

Higher Tier textbook, section 13.1, page 256.

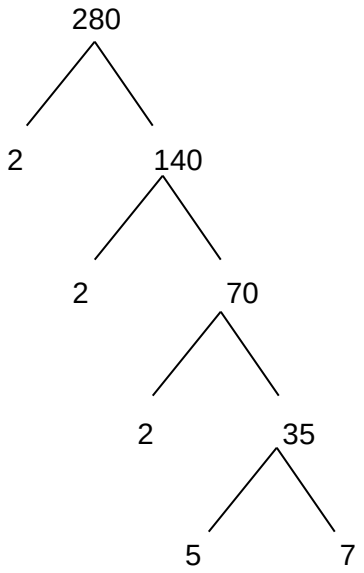
Example 4: Using prime factors to find the highest common factor

- **Find the HCF of 150 and 280.**

a) Using prime decomposition, find the prime factors of both numbers and write these out in index form:



$150 = 2 \times 3 \times 5^2$



$280 = 2^3 \times 5 \times 7$

b) Now we look for a prime number that is in both lists, and we underline the lowest power of this number. 2 and 5 are in both of our lists. The lowest power of 2 is 2^1 and the lowest power of 5 is 5^1.

c) Multiply these two numbers together to find the HCF. 2 x 5 = 10, **so the HCF of 150 and 280 is 10.**

If we can work out the prime decomposition of numbers then it does save the time that it takes to write out the list of factors. This can be quite time consuming for large numbers.

2.7 Finding the LCM using prime factors

Example 5: Using prime factors to find the least common multiple

- **Find the LCM of 150 and 280.**

a) The first step is the same as before. Find the prime factors of both numbers. We already have these:

$150 = 2 \times 3 \times 5^2$ $280 = 2^3 \times 5 \times 7$

b) This time we want the highest power of each prime number in either list; they don't have to be in both lists.

The highest power of 2 is 2^3. The highest power of 3 is 3^1 The highest power of 5 is 5^2, and the highest power of 7 is 7^1.

c) Multiply these numbers together to find the LCM:

$2^3 \times 3 \times 5^2 \times 7$ **= 4,200**

Important point:

If you find the two methods confusing, remember that a factor (even the highest common factor) can't be bigger than the original numbers and a multiple cannot be smaller than the original numbers.

Things to do: (FOUNDATION TIER)

Work through section 13.2 on page 276 of the Foundation Tier textbook. Note the use of Venn diagrams to find the HCF and LCM. Now work through exercise 13.2 S on page 277. The applications are shown on page 278. Work through these and then complete exercise 13.2A on page 279.

If you are not familiar with Venn Diagrams, you can find out more about them by visiting this webpage:

MathsIsFun.com: Sets and Venn diagrams

Things to do: (HIGHER TIER)

Work through section 13.1 of the Higher Tier textbook, which starts on page 256. In this section of the textbook we can see how Venn diagrams can be used to work out the HCF and LCM of a pair of numbers. You can use this method if you find it easier.

If you are not familiar with Venn Diagrams, you can find out more about them by visiting this webpage:

MathsIsFun.com: Sets and Venn diagrams

You should also work through exercise 13.1S on page 257 of your textbook, and exercise and 13.1A on page 259.

In a nutshell

In this section, we looked at how to find the highest common factors and the least common multiple of two or more numbers, and we found out how finding the prime factors can help us.

Recommended reading

DLC resources

DLC Library: Log in to your Learner Account and click on 'Library' to view various resources to help you with your learning

DLC Forum: https://forum.distancelearningcentre.com/index.php

Websites

BBC GCSE Bitesize: Number